

Raphaël Beuzart (IAS)

The local Gan-Gross-Prasad conjectures (18/06, 2 pm & 19/06, 10.45am, 2 pm & 20/06, 9 am)

The purpose of this series of lectures is to present a proof of the local Gan-Gross-Prasad conjectures (Bessel case) over p -adic fields. These conjectures give a very precise description of some branching-laws between orthogonal or unitary groups. These predictions are based on the local Langlands correspondence seen as a way to parametrize irreducible representations of the relevant groups. Waldspurger (for tempered representations) and then Mœglin-Waldspurger (for more general representations) gave a complete proof of the conjecture for orthogonal groups. I will present a proof for tempered representations of unitary groups that follows closely the pioneering work of Waldspurger on the subject.

Wee-Teck Gan (NUS) & Atushi Ichino (Kyoto)

Gross-Prasad conjecture and local theta correspondence

The series of 4 talks by Gan and Ichino is devoted to the proof of the Fourier-Jacobi case of the local Gross-Prasad conjecture for unitary groups, by reducing it to the Bessel case via local theta correspondence. A brief description of the contents of each talk is:

Talk 1 (Gan, 23/06, 11.30 am): we formulate the Fourier-Jacobi case of the GP conjecture and explain how it can be deduced from the Bessel case if one understands the local theta correspondence for $U(n) \times U(n+1)$ and $U(n) \times U(n)$ completely. Indeed, we shall see that knowing the $U(n) \times U(n+1)$ theta correspondence is enough.

Talk 2 (Ichino, 23/06, 3.15 pm): we introduce the local theta correspondence and describe two conjectures of D. Prasad concerning the $U(n) \times U(n)$ and $U(n) \times U(n+1)$ theta correspondence. We also discuss how various representation theoretic invariants behave under the local theta correspondence.

Talk 3 (Gan, 24/06, 11.30 am): we describe the proof of Prasad's conjecture for the $U(n) \times U(n+1)$ theta correspondence in the case of discrete series representations. This uses a global-to-local argument which relies on Arthur's multiplicity formula for unitary groups.

Talk 4 (Ichino, 24/06, 2.00 pm): we describe how the tempered case of Prasad's conjecture can be proved by an explicit construction of local theta lifting and a detailed understanding of the normalised intertwining operators and their local intertwining relations.

Benedict Gross (Harvard)

Lecture 1. An introduction to the GGP conjectures (18/06, 9 am). I will give an introduction to the local and global conjectures for orthogonal groups, focusing on the cases of low rank which were the first to be studied.

Lecture 2. A statement of the conjectures in the orthogonal case (18/06, 10.45 am). I will state the local and global conjectures for restriction of representations of orthogonal groups. The local conjectures involve symplectic root numbers and the global conjectures involve the central values of L-functions.

Lecture 3. The first derivative (19/06, 9 am). I will present some conjectures on the first derivative of the global L-function, in the case when the sign of the functional equation is equal to -1 . In this case, one replaces the product of period integrals on the space of automorphic forms with the height pairing on the group of cycles on a Shimura variety.

Micahel Harris (IMJ)

Some arithmetic applications of the Ichino-Ikeda conjecture (27/06, 9 am)

The Ichino-Ikeda conjecture, in the version of N. Harris, is a formula relating special values of L-functions of certain automorphic representations to explicit period integrals on unitary groups. When the automorphic representations are attached to coherent cohomology of Shimura varieties, the periods can be given motivic interpretations. I will discuss some applications to special values of L-functions, in the spirit of Deligne's conjecture.

If time permits, I will also describe some potential applications in analytic number theory, and to p-adic L-functions.

Ben Howard (Boston College)

Modularity of generating series of cycles I and II (26/06, 4.30 pm & 27/06, 2 pm)

In the first talk I will sketch a proof of the modularity of a generating series of arithmetic divisors on a unitary Shimura variety. In the second talk I will describe two applications. The first application is to a Gross-Zagier theorem for higher weight modular forms. The second application is the proof of new cases of Colmez's conjecture on the Faltings heights of CM abelian varieties.

Hervé Jacquet (Columbia)

Two problems in the theory of the relative trace formula (25/06, 11 am).

Let H be a spherical subgroup of G reductive. If we integrate the kernel of the trace formula over the product $H \times H$ the contribution of the continuous spectrum may involve improper integrals. We discuss a simple example. In other situations, it is natural to try to compare this relative trace formula with an ordinary trace formula on a different group G' . It turns out that this is not possible because we need to introduce a weight factor. We discuss a conjectural example.

Steven Kudla (Toronto)

Modularity of generating series for special cycles (26/06, 11 am & 2 pm)

In these lectures, I will survey some of the ideas and results about the modularity of generating series for special cycles on the Shimura varieties for unitary groups of signature $(n-1, 1)$ and for orthogonal groups of signature $(n-1, 2)$.

In the first lecture, I will review the definitions of these cycles via complex uniformization and via moduli. Then I will concentrate on the case of 0-cycles and the conjectural relations to central values and derivatives of Eisenstein series, highlighting some important open problems.

In the second lecture, I will discuss the case of generating series valued in Chow groups beginning with Borcherds' proof in the case of divisors and, in particular reviewing Borcherds' construction of meromorphic modular forms via the regularized theta lift. Then I will discuss the work of Wei Zhang and recent results of Raum and Bruinier concerning cycles of higher codimension. Finally, following Bruinier, I will explain how Green functions for special divisors can be constructed by applying Borcherds' construction to harmonic weak Maass forms.

Dipendra Prasad (TIFR)

Ext Analogues of Branching laws (23/06, 2 pm)

The decomposition of a representation of a group when restricted to a subgroup has been studied in many instances. In this lecture, we will look at a variation on these questions involving concepts in homological algebra which aims to find simpler and more flexible theorems.

Michael Rapoport (Bonn)

Around the Arithmetic Fundamental Lemma of Wei Zhang (26/06, 3.15 pm & 27/06, 11 am)

The AFL arises in the relative trace formula approach to the Arithmetic Gan-Gross-Prasad Conjecture. It is a conjectural relation between the derivative of an orbital integral and an intersection number on a formal moduli space of p -divisible groups. I will explain the statement of the AFL and report on its status. I will also report on ongoing work of B. Smithling, W. Zhang and myself on variants of AFL involving ramification.

Yiannis Sakellaridis (Rutgers)

Lecture 1. "Local periods and the Plancherel formula" (23/06, 9 am): The global period integrals of the Gan-Gross-Prasad conjectures conjecturally factorize as Euler products of local "periods" given, à la Ichino-Ikeda, by integrals of matrix coefficients. I will explain how to understand these local periods in terms of the local Plancherel formula of the corresponding homogeneous space. This leads to a proof of their positivity when the given representation is distinguished.

Lecture 2. "Periods and distinction for spherical varieties" (24/06, 9 am): The conjectures of Gan-Gross-Prasad fit into the general framework of spherical varieties, and Jacquet's concept of "distinction". I will explain this framework, including, if time permits, methods more general than "periods" such as the Rankin-Selberg method and a speculative attempt for a vast generalization, including Ngô's recent foray into reductive monoids.

Lecture 3. "In search of the L-function" (25/06, 9 am): There is very little understanding about the (special value of an) L-function associated to each period. In anticipation of a nicer answer, I will present a combinatorial recipe that relates the L-function to geometric invariants of the relevant spherical variety.

Binyong Sun (Chinese Academy of Science)

Arithmetic of critical values of some Rankin-Selberg L-functions for $GL(n) \times GL(n-1)$ (26/06, 9 am)

Abstract: Following the works of Kazhdan-Mazur-Schmidt, Schmidt, Kasten-Schmidt, Raghuram-Shahidi, Januszewski, Grobner-Harris, Raghuram, and etc., I will explain the algebraicity of critical values of certain Rankin-Selberg L-functions for $GL(n) \times GL(n-1)$, and in the ordinary case, the p -adic boundedness of their twists by finite order Hecke characters. I will also explain the archimedean non-vanishing hypothesis which is vital for the arithmetic study of the critical values.

Shunsuke Yamana (IMJ)

Periods of residual automorphic forms (23/06, 4.30 am)

I gave the same talk at the BIRS workshop on "The future of trace formulas".

The people who were there are suggested to skip my talk.

I will discuss periods of residual automorphic forms by giving as many examples as possible. The distinguished residual automorphic representations are classified for $(GL(n+1) \times GL(n), GL(n))$, $(GL(n, E), GL(n, F))$, $(GL(n) \times GL(n) \times GL(n), GL(n))$, where E is a quadratic extension of F and ${}^{\wedge}GL(n)$ is the double cover of $GL(n)$. I here consider H -periods of automorphic forms on G for the pairs (G, H) .

Wei Zhang (Columbia)

RTF and the global conjecture for unitary groups (18/06, 3.45 pm & 19/06, 3.45 pm & 20/06 10.45 am, 2 pm)

In this short course we will present the approach of relative trace formula (RTF) of Jacquet and Rallis to the global Gan-Gross-Prasad for unitary groups, and to the Waldspurger-Ichino-Ikeda type formulae. The main ingredients are the relevant fundamental lemma, the smooth transfer, and a local character identity.

Michal Zydor (IMJ)

The relative trace formula of Jacquet-Rallis (24/06, 3.15 pm)

A truncation process à la Arthur for the relative trace formula of Jacquet-Rallis will be presented. I will describe spectral and geometric expansion and its properties for both unitary and general linear group. If time permits, I will discuss some cases in which we can get a more precise formula.